On the informative content of dynamic Hurst exponents: a comparison among different techniques

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Abstract: In this paper we explore the informative content of time dependent Hurst exponents, separately introduced in recent years by various authors. To such aim, we have considered two different estimation methods: the Detrended Moving Average (DMA) of Carbone & Stanley (2004), and the multiresolution method of Fleming & Jubb (2001). Those techniques have been applied on a set of financial timeseries, sampled at various time frequencies, with daily units assumed as upper border frequencies, and the results compared with those obtained on artificially generated monofractal series. At the present stage, our major findings may be summarised as follows: (a) by comparison with the other technique, the DMA tends to underestimate Hurst exponents at every time frequency; (b) the local Hurst exponents, evaluated on observable data, reveal stronger time variability than in artificial monofractal series.

Keywords: Dynamic Hurst exponents, daily and intraday timeseries, monofractal artificial timeseries.

1 Literature review

As widespread known, the rescaled range analysis (or R/S analysis) recalls a method originally introduced by the hydrologist Harold Hurst (Hurst, 1951). In its classical form, given the series of returns $\{X_j\}_{j=1}^{n}$ of length n, the method provides the statistics:

$$R/S(n) = \frac{1}{s_n}\left\{\max_{1\le k\le n}\left[\sum_{j=1}^{k}(X_j-\bar{X}_n)\right] - \min_{1\le k\le n}\left[\sum_{j=1}^{k}(X_j-\bar{X}_n)\right]\right\} \qquad (1)$$

where s_n is the sample standard deviation, while the first and second terms on the right hand side of Eq.(1) refer, respectively, to the maximum and the minimum of the partial sums of the first k deviations of X_j from the sample mean.

Within an historical perspective, the relevance of the R/S analysis to detect long range dependence was suggested by (Mandelbrot & Wallis, 1969), who highlighted the existence in random processes of a scaling relationship between the rescaled range and the number of observations n:

$$R/S(n) \propto nH \qquad (2)$$

where H is the so called *Hurst exponent*. In addition, those authors provided evidence, via Montecarlo simulations, that, for sufficiently large n, a proxy of the Hurst coefficient might be given through the regression of the logarithm of $R/S(n)$ against $log(n)$. In the same way, they gave proof that $H = 0.5$ can be associated to sample paths generated by a Geometric Brownian Motion, whereas $H > 0.5$ usually accounts for underlying persistent, long memory processes and, conversely, $H < 0.5$ is generally intended as a signal of antipersistent data.

Despite of its broad application to financial timeseries, however, various authors (such as, among the others: Lo, 1991, Moody & Wu, 1996) pointed on the sensitivity of H to short range dependence; additionally, it has been argued in (Karuppiah & Los, 2005) that the Hurst exponent, in its original version, as well as in its further developments, can only provide a measure of the global behaviour of observable data. This means that H is not able to give any explanation about local patterns (that are the rule, and not the exception, in financial markets); this, in turn, is potentially dangerous, since it can lead to misspecifications and fallacious conclusions in data modeling.

Such lack of reliability of the global Hurst exponent has been recently bypassed in a number of contributions, that suggested alternative techniques for the estimation of a pathwise version of H. Although almost three techniques appear to be particularly appealing and promising, that is, in detail, the dynamic technique of (Bianchi, 2005), the Detrended Moving Average (DMA) of (Carbone & Stanley, 2004), and the multiresolution method of (Fleming & Jubb, 2001), at the present stage we are able to provide results on the latter two. In particular, we have applied those cited methods to a set of financial series, sampled at different time frequencies (5 minutes, 30 minutes, one hour, four hours, and one day). Our aim is twofold. First of all, we are interested to explore the informative content of dynamic Hurst exponents $H(t)$ at various time scales, monitoring if they are able to capture additional richness of information from data, and how much such capability can vary, when the time unit is changed (from high frequency to daily). Additionally, we have studied to what extent the results provided by such methods are somewhat equivalent or completory one to each other. To such purpose, we have compared the results on observable data with those obtained with artificially generated monofractal timeseries, used in this context as benchmark series.

Within this frame, our study is structured as follows. Section 2 will give basic details about the techniques we have employed. In Section 3, after introducing the data used in our simulation, we will provide and discuss our results. Finally, Section 4 will conclude the paper.

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