CLASSIFYING THE ITALIAN PENSION FUNDS VIA GARCH DISTANCE

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ABSTRACT: The adoption of pension funds in the Italian social security policy has increased the offer of several investment funds. The workers have to decide what kind of investment to perform, the funds having a different composition and a subsequent different degree of risk. In this paper we propose to use a distance between GARCH models as a measure of different volatility of some funds, with the purpose to classify a set of funds in terms of different risk. An application on thirteen Italian funds is performed.

KEYWORDS: Agglomerative algorithm, Cluster analysis, Maximum risk profile

1 Introduction

The main aim of a pension fund is to rise workers' savings and to invest them according to an accurate policy of asset allocation, in order to give back the hoarded capital as a life annuity. Therefore, the most important index pension fund refers to the global asset return since it influences both periodic contributions and future benefits. For this reason the valuation of a pension fund is often related to the performance rather than to the risk level of asset portfolio. According to Ryan and Fabozzi (2003) the USA pension shortfalls after 2001 are not the consequence of a poor market performance, but the inevitable result of actuarial and fiscal accounting practices. In fact, in the long term the expected asset growth must be higher for equities than for bonds. Bader (2003) has an opposite position and argues that risky investment like equity has no place in portfolios pension funds. Such a rule should be applied particularly for pension funds when they concern public and not complementary pension. In Italy the pension plans of private sector workers are run by private funds controlled by government. Trudda (2005) studies actuarial balances and dynamics for independent consultants pensions (ICP) plans. The analysis of each random variable shows that a marginal increase of global asset return gives an important reduction of default probability. In his analysis Trudda (2005) puts in evidence that this kind of funds belongs to the first pillar pensions group and they can not be assimilated as complementary pensions fund random variables due to technical and social reasons. Therefore the global asset return analysis should refers to a risk benchmark rather than a return benchmark, recognizing the maximum risk profile. The performance of these ICP funds is variegated depending on their asset composition. Some of them have only real or bond investment while some others have high stock market component. The 2006 annual report on the condition of public and private welfare Italian Agency affirms that most of these funds are now shifting on higher risk level portfolios. The Agency stresses the social rather than speculative function of such funds which valuate the maximum risk profile in function of the liability for pension payments. For these reasons an important task is to classify the different funds with respect to their different real degree of risk. In statistical terms we have to follow the dynamics of the time series referred to the single funds and to perform an appropriate cluster analysis. In this paper we use an agglomerative algorithm proposed by Otranto (2004) based on the distance between couple of time series following GARCH models. This distance is an extension of the well known distance between invertible ARMA models proposed by Piccolo (1990). The logic of this approach in this framework is explained by the link existing between volatility and investment risk. High volatility periods correspond to turmoil phases in the dynamics of the time series studied; if we represent the conditional variance of the series with GARCH models, the GARCH structure will reflect the behavior of the volatility. In other words, similar GARCH models represent similar volatility behavior and similar investment risks. The comparison of the GARCH models underlying the series relative to different pension funds, provides a classification of the funds based on different degrees of risk. For this purpose we need a benchmark time series; we choose the fund which potentially presents the most hazardous global asset risk profile.

In the next section we briefly recall the GARCH distance proposed by Otranto (2004) and the agglomerative algorithm used; in section 3 we show an example of classification based on this methodology using a risk banchmark to mark a maximum risk profile as suggested by Trudda (2005).

2 Distance between GARCH models

Let us consider two time series following the models ($t = 1, ..., T$):

$$\begin{aligned} y_{1,t} &= \mu_1 + \varepsilon_{1,t}, \\ y_{2,t} &= \mu_2 + \varepsilon_{2,t}; \end{aligned} \tag{1}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are disturbances with mean zero and time-varying variances. We suppose that the conditional variances $h_{1,t}$ and $h_{2,t}$ follow two different and independent GARCH(1,1) structures:*

$$\begin{aligned} Var(y_{1,t}|I_{1,t-1}) &= h_{1,t} = \gamma_1 + \alpha_1 \varepsilon^2_{1,t-1} + \beta_1 h_{1,t-1} \\ Var(y_{2,t}|I_{2,t-1}) &= h_{2,t} = \gamma_2 + \alpha_2 \varepsilon^2_{2,t-1} + \beta_2 h_{2,t-1} \end{aligned} \tag{2}$$

where $I_{1,t}$ and $I_{2,t}$ represent the information available at time t and $\gamma_i > 0$, $0 < \alpha_i < 1$, $0 < \beta_i < 1$, $(\alpha_i + \beta_i) < 1$ ($i = 1, 2$).

*In our applications we have noted that the $GARCH(1,1)$ models fit the series analysed, so we explain the idea of distance only for this case. For the extension to a generical $GARCH(p,q)$ model see Otranto(2004).

As well known, the squared disturbances in (2) follow ARMA(1,1) processes:

$$\varepsilon_{i,t}^2 = \gamma_i + (\alpha_i + \beta_i)\,\varepsilon_{i,t-1}^2 - \beta_i \left(\varepsilon_{i,t-1}^2 - h_{i,t-1}\right) + \left(\varepsilon_{i,t}^2 - h_{i,t}\right),\ i = 1,2 \qquad (3)$$

where $\varepsilon_{i,t}^2 - h_{i,t}$ are mean zero errors, uncorrelated with past information. As a matter, the two GARCH(1,1) models can be compared in terms of the distance measure proposed by Piccolo (1990). Otranto (2004) shows that this distance, in the GARCH(1,1) case, can be expressed as:

$$d = \left[\frac{\alpha_1^2}{1-\beta_1^2} + \frac{\alpha_2^2}{1-\beta_2^2} - \frac{2\alpha_1\alpha_2}{1-\beta_1\beta_2}\right]^{1/2} \qquad (4)$$

The hypothesis of null distance can be easily tested noting that (4) is equal to zero if and only if $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. This hypothesis is verified using the statistic:

$$W = (\boldsymbol{A}\widehat{\boldsymbol{\theta}})'(\boldsymbol{A}\widehat{\boldsymbol{\Lambda}}\boldsymbol{A}')^{-1}(\boldsymbol{A}\widehat{\boldsymbol{\theta}}) \qquad (5)$$

where $\widehat{\boldsymbol{\theta}}$ represents the maximum likelihood estimator of $\boldsymbol{\theta} = (\alpha_1, \beta_1, \alpha_2, \beta 2)'$, whereas $\widehat{\boldsymbol{\Lambda}}$ is the estimated covariance matrix of $\widehat{\boldsymbol{\theta}}$ and $\boldsymbol{A} = [\boldsymbol{I}_2, -\boldsymbol{I}_2]$, where $\boldsymbol{I}_2$ is the 2×2 identity matrix. The statistic (5) follows the chi-square distribution with 2 degree of freedom.

This distance can be used to cluster *n* time series in homogenous groups, having similar GARCH structure. In other words we insert the series with not significant distance in the same cluster. For this purpose an agglomerative algorithm could be used; following Otranto (2004) it can be developed in the following steps:

1. choose an initial benchmark series;
2. insert in the group of the benchmark series all the series with a distance from it not significantly different from zero (using the statistic (5));
3. select the series with the minimum distance from the benchmark significantly different from zero; this series will be the new benchmark;
4. insert in the second group all the remaining series with a distance from the new benchmark not significantly different from zero;
5. repeat steps 3 and 4 until no series remain.

Note that the number of groups is not fixed a priori or chosen after the clustering, but it derives automatically from the algorithm.

Clearly, to classify the series we need a starting point, in the sense that the result will be different, changing the series adopted as initial benchmark. For our purposes we choose the fund having the highest potential risk, being mainly constituted by shares.

3 A classification of funds

We consider thirteen time series from Nov.1995 to Dec.2000 (daily data; 1290 observations; sources: Independent Consultants Pension Funds (ICPF) balance sheet, Complementary Pension Funds (CPF) balance sheet, and Supplements to the Bank of Italy Statistical Bulletin) relative to some basis and complementary Italian pension funds. The funds are classified, respect to their asset portfolio composition, as follow:

A = CPF with a mixed (bond and stock funds) financial products; B = ICPF with asset divided in investment properties and Italian government bonds; C = CPF with real estate investments, 10 years duration benchmark bonds and small stock component; D = ICPF with real estate and 3 years duration benchmark bonds; E = ICPF with liquidity, properties and 30 years duration benchmark bonds; F = CPF with 3 years duration benchmark bonds and small stock component; G = ICPF with prevalence of investments in government bonds funds; H = Investment fund with interest rate indexation benchmark; I = Investment fund with interest rate indexation benchmark; J = ICPF with prevalence of bonds investments; K = CPF with prevalence of stock funds investments; L = CPF with prevalence of bond funds investments; M = CPF with mixed, bond and stock funds, investments.

For each time series we have estimated a GARCH(1,1) model as (2). Assuming K, which is theoretically the most risky fund, as initial benchmark, we obtain the distance (4) from K shown in Table 1.

	A	B	C	D	E	F	G	H	I	J	L	M
K	0.01	0.17	0.09	0.13	0.22	0.06	0.12	0.16	0.11	0.11	0.09	0.02

Table 1. *GARCH(1,1) distance of twelve funds from fund K*

Applying the algorithm illustrated in the previous section, we obtain the following clusters:

CLUSTER 1	CLUSTER 2	CLUSTER 3	CLUSTER 4	CLUSTER 5
K A M	F L C	I	J G D H	B E

The results confirm that the lower risk profile is related to the basis pension funds that invest mostly in properties, real estate and bond funds maintaining anyway a minimum return. On the other hand the funds that are in the most risky group are the complementary pension funds with high tendency on stock market investments.

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