A BIOMETRIC RISKS ANALYSIS IN LONG TERM CARE INSURANCE

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ABSTRACT: Long Term Care (LTC) covers are insurance products for which the risk is difficult to assess, since there is a lack of reliable statistical data that can be used for the purposes of rating. Also experience data about LTC claims are rather inadequate. Moreover, due to LTC risk features, LTC covers are considerably influenced by future mortality and disability trends. This paper deals with problem of analysing uncertainty arising from both mortality and disability risks, expressed by means of the loss function. To this purpose some mortality and disability projected scenarios are taken into account. The probabilistic structure adopted is consistent with multiple state models, based on a time-continuous Markov chain.

KEYWORDS: Long Term Care covers, Biometric risks, Demographic trends, Loss function for LTC covers.

1 Introduction

LTC insurance purpose is to offer some form of protection against costs arising from future deterioration in health. The most critical issues in defining LTC risk arise from lack of reliable experience data, uncertainty about future trends in mortality and disability in the older groups, adverse selection and moral hazard. Moreover, in Italy LTC covers are recent products (their first issue on the Italian market was in 1997), so insurance experience data about LTC claims are rather inadequate to risk assessment.

This problem could be face taking into account population health data. Nevertheless, population data are usually expressed as prevalence rates, while insurance pricing requires incidence rates. They could be obtained starting from prevalence rates on the basis of a set of adequate assumptions.

Apart from biometric risks, LTC covers are mainly affected by investment risks, but our analysis focuses exclusively on biometric risks deriving from disability and life duration. Specifically, the uncertainty in future trends of mortality among both healthy and disabled lives and in future trends of disability at old ages generates the risk of systematic deviations in the biometric functions adopted in pricing and reserving.

2 Multiple state model

LTC insurance is usually modelled according to a multiple state model (a description of such models can be found in Haberman & Pitacco (1999)).

Let $S(t)$ represent the random state occupied by the insured at time t, for any $t \geq 0$, where t represents the policy duration and 0 is the time of entry. The possible realizations of $S(t)$ are: 1 = "active" (or healthy), 2 = "LTC disabled", 3 = "dead". Only one level of disability is considered. We assume $S(0) = 1$. Moreover, because of the usually chronic character of disability, the possibility of recovery from the LTC state is disregarded; therefore transition $2 \rightarrow 1$ is ignored.

The graph in Figure 1 illustrates the set of states occupied by the policyholder in a time instant and the set of possible direct transitions between states.

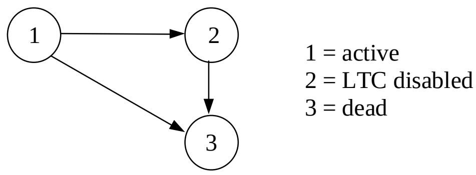

Figure 1. *A multiple state model for LTC insurance with one level of disability.*

Let us assume that the stochastic process $\{S(t);\ t \geq 0\}$ is a time-continuous, time inhomogeneous, three states Markov chain. Let us define transition probabilities:

$$P_{ij}(t,u) = \Pr\{S(u) = j \,|\, S(t) = i\} \qquad 0 \leq t \leq u, \qquad i, j \in \{1,2,3\} \tag{1}$$

and transition intensities:

$$\mu_{ij}(t) = \lim_{u \to t} \frac{P_{ij}(t,u)}{u - t} \qquad t \geq 0, \quad i, j \in \{1,2,3\},\ i \neq j \tag{2}$$

Following the Transition Intensities Approach, transition intensities have to be assigned to derive the transition probabilities according to the model depicted in figure 1.

3 Estimation of transition intensities

Transition intensities defined in (2) are estimated according to a set of adequate assumptions, starting from ISTAT data (see ISTAT (2000)). The first step is to taking into account the following Kolmogorov forward differential equations:

$$\frac{d}{dt}(P_{12}(z,t)) = P_{11}(z,t)\,\mu_{12}(\mathrm{t}) - P_{12}(z,t)\mu_{23}(\mathrm{t}) \qquad 0 \le z \le t \tag{3}$$

$$\frac{d}{dt}(P_{11}(z,t)) = - P_{11}(z,t)\left(\mu_{12}(\mathrm{t}) + \mu_{13}(\mathrm{t})\right) \qquad 0 \le z \le t \tag{4}$$

From (3) transition intensities $\mu_{12}(t)$ can be obtained as a function of $P_{11}(z,t)$, $P_{12}(z,t)$ and $\mu_{23}(t)$:

$$\mu_{12}(\mathrm{t}) = \frac{\dfrac{dP_{12}(z,t)}{dt} + P_{12}(z,t)\mu_{23}(\mathrm{t})}{P_{11}(z,t)} \qquad 0 \le z \le t \tag{5}$$

$P_{11}(t,t+1)$, $P_{12}(t,t+1)$ and $P_{13}(t,t+1)$ have been calculated for each integer t starting from prevalence rates of LTC disabled (see ISTAT (2000)) and Italian Life Table data (SIM 1999). In order to obtain continuous functions for $P_{11}(z,t)$ and $P_{12}(z,t)$, their annual values are interpolated using the least squares method with GM (0,4) Gompertz-Makeham functions. In order to find $\mu_{23}(t)$ intensities, LTC disabled mortality is assumed to be related to active mortality:

$$P_{23}(t,t+1) = k(t) \cdot P_{13}(t,t+1) \tag{6}$$

where values of the time-dependent factor $k(t)$ come from experience data of an important reinsurance company. The mortality intensities $\mu_{23}(t)$ can be estimated according to the following approximation formula (see Pitacco (2000)):

$$\mu_{23}(t) = -0.5[\,\ln(1 - P_{23}(t-1,t)) + \ln(1 - P_{23}(t,t+1))] \tag{7}$$

Moreover, from (4) actives mortality intensities are computed:

$$\mu_{13}(\mathrm{t}) = -\frac{d\ln P_{11}(z,t)}{dt} - \mu_{12}(\mathrm{t}) \qquad 0 \le z \le t \tag{8}$$

Results show that actives’ mortality can be well approximated by the Weibull law, while transition intensities $\mu_{12}(t)$ show an exponential behaviour so the Gompertz law has been used:

$$\mu_{13}(t) = \frac{\beta}{\alpha}\left(\frac{x+t}{\alpha}\right)^{\beta-1} \qquad \alpha, \beta > 0 \tag{9}$$

$$\mu_{12}(t) = \eta \cdot e^{\lambda(x+t)} \qquad \eta, \lambda > 0 \tag{10}$$

4 Demographic scenarios

When LTC annuity benefits are concerned a key point in actuarial evaluations is to measure the risk coming from the uncertainty in disability duration, i.e. the time spent in LTC disabled state. To this purpose it is necessary to make some assumptions about the link between mortality and disability. With regard to such link three main theories have been formulated (for an overall review see Olivieri and Pitacco (2001)): pandemic, equilibrium and compression theory. Therefore, some reliable scenarios have been defined according to these theories, including projection of both mortality and disability.

A basic scenario is based on ISTAT health and mortality data, while projected scenarios have been developed relating to their impact on the time spent in both healthy and LTC disabled states. In projected scenarios different set of Weibull parameters (α, β), have been evaluated consistently with ISTAT projections (low, main and high hypothesis, ISTAT (2002)); while Gompertz parameters (η, λ) has been assessed to represent increase or decrease in disability trend.

5 Risk analysis

Let us consider one cohort of policyholders homogenous with respect to age at policy issue, year of entry, benefits amount and risk class. The following benefits are involved: a deferred annuity paid when the insured is healthy, a deferred enhanced annuity when the insured is in the LTC state and a lump sum benefit on death occurring before the deferment period.

Both single and annual constant premiums are computed according to the equivalence principle. In order to perform a risk analysis the random present value and loss function are calculated under both scenario and stochastic approach.

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