THE FAIR VALUE OF THE INSURED LOAN PORTFOLIO SCHEDULED AT VARIABLE INTEREST RATES.

Mariarosaria Coppola*
Valeria D'Amato**
Marilena Sibillo***

*Università degli Studi di Napoli Federico II
e-mail: m.coppola@unina.it

**Università degli Studi di Napoli Federico II
e-mail: valeriadamato@virgilio.it

***Università degli Studi di Salerno
e-mail: msibillo@unisa.it

ABSTRACT: The paper concerns the case of the insured loan based on an amortization schedule at variable interest rates. Basing on the cash flow structure, the aim is to evaluate the mathematical provision of a portfolio in a fair value approach. In this environment, the complexity of a life insurance contract management practically involves the choice of the most suitable mortality table and discounting process; in the paper the tool is treated in a stochastic scenario for interest rates and in random hypotheses for the mortality rates. The amortization schedule used for the loan repayment is considered at variable interest rates, hooked at opportune rate indexes. A numerical application of the model is presented and a comparison between the behaviour of the fair values of the insured loan portfolio reserve in the two cases of an amortization schedule at fixed and at variable interest rates is reported. The fair reserve sensitivity to the changes of the amortization interest rate is studied and showed with illustrations.

KEYWORDS: **Fair value, insured loan, variable interest rates, Cox-Ingersoll-Ross model.**

§1. Introduction

The International Boards working in the life insurance business accounting field, in order to define a proper assessment of the risk in the solvency tool and to realize a homogeneous and correct information of the insurance company activity among different companies of different countries, indicates the reserve quantification as a mark-to-market valuation of the insurance liabilities, that is the so-called *fair value.* The *financial risk*, referred to the uncertainty due to movements of the interest rates and the *demographic risk*, represented by the uncertainty both accidental and systematic in the insureds future lifetime, are the two main risk sources affecting the portfolio evaluations.

The paper concerns the insured loan, an insurance product strictly connected to the common financial operation of a loan repaid by the amortization method. The aim of the contract is to square the debt in the case of the insured-borrower predecease. Nowadays different kinds of basic loans are offered in the market, and most of them are repaid at variable interest rates with constant instalment at the end of each period. In these basic contractual hypotheses, the paper presents a model for the fair valuation of the insured loan portfolio reserve, taking into account the two risk sources above introduced. As in [5] [3] [4], the approach we follow is the fair value calculation at current values, using the best estimate of the current interest and mortality rates.

From the strictly actuarial point of view, this is the framework of a n-year term life insurance with decreasing insured sums.

§2 The amortization schedule.

The amortization schedule we assume is based on constant periodic instalments calculated at variable interest rates, settled according to the level of an interest rate index fixed in the contract and observed the day before the instalment maturity. Most of the contracts refer to the Euribor rate, which is a mean of the interest rates applied by the European banks among themselves, monitored and published by the BCE.

On the basis of the amortization method at variable interest rates (cf. [6], in the insured loan contract the insurer will repay to the lender the obligations due by the borrower, if this one dies during the contract duration; at time *h* they consist in the outstanding balance D_{h-1} at time h-1 plus the interest on this sum for the period *h-1,h* (cf. [4]). The value B_h of the benefit payable at time *h (h= 1,2,...n)* if the insured-borrower aged x at issue dies during the *h*-th year and the probability of this event are respectively:

$B_h = D_h(1+i^*)$ $\qquad$ ${}_{h-1/1}q_x$

where i^* usually represents the Euribor observed the day before the maturity *h.*

§2 The fair value of the insured loan portfolio.

We consider a portfolio of *c* homogeneous insured loans repayable at variable interest rates. Let us suppose that each contract is issued on an insured aged x, with premiums payable at the beginning of each period till the insured is alive or up to *m* payments *(1* $\le m \le$ *n)*, and benefit payable at the end of the period of the insured's death, if this event occurs before *n*. Considering that the benefit is the sum of the outstanding balance at the beginning of the period and the periodic interest due to that amount and indicating by k_x the curtate future lifetime of the insured aged x at issue, within a deterministic scenario and in the case of anticipated premium payments, the flow at time h is given by the following scheme:

$$X_h = \begin{cases} -_{/m}P_{x,h+1} & k_x \geq h & 0 \leq h \leq m-1 \\ 0 & k_x \geq h & h \geq m \\ D_{h-1}(1+i^*) & h-1 \leq k_x < h & 1 \leq h \leq n \end{cases}$$

where $_{/m}P_{x,h+1}$ is the *(h+1)-th* premium payable at the beginning of the *h-th* year.

The generic cash flow connected to the entire portfolio is given by:

$$f_0 = -c\,_{/m}P_{x,1} \qquad \text{if } h=0$$

$$f_h = -_{/m}P_{x,h+1} \quad n_h + D_{h-1}(1+i^*)(n_{h-1} - n_h) \qquad \text{if } h = 1,2,\ldots n$$

where n_h is the number of survivors at time h.

Introducing a stochastic scenario, let us consider the probability space $\{\Omega, \Im, \wp\}$ originated by the two probability spaces: $\{\Omega, \Im', \wp'\}$ and $\{\Omega, \Im'', \wp''\}$, referred respectively to the financial and the demographic events (cf. [3]). Under the usual hypotheses of the competitive market (cf.. [2]), we indicate by:

- $\tilde{N}_h$ the random variable representing the number of survivors at time h belonging to the group of those, among the c initial insureds at time 0, are living at time t,
- *v(t,h)* the stochastic present value at time t of one monetary unit at time h,
- F_h the stochastic flow at time h
- L_t the stochastic loss in t of the portfolio of c contracts,
- $K_{x,t}$ the curtate future lifetime at time t of the insured aged x at issue,

According to a risk neutral valuation, we obtain the stochastic loss at time t in its fair value form replicating the stochastic flow F_h at time h ($h>t$) by a trading strategy, being $\tilde{N}_h \;\, _{/m}P_{x,h+1}$ the number of units of Zero Coupon Bonds issued at time t and maturing at time h:

$$\mathrm{E}[\mathrm{L_t}/\Im_t] = \mathrm{E}\left[\sum_{h=t+1}^{n}\left[\tilde{N}_h \;\, _{/m}P_{x,h+1} + \left(\tilde{N}_h - \tilde{N}_{h-1}\right)D_{h-1}\left(1+i^*\right)\right]v(t,h)/\Im_t\right] \tag{1}$$

The expected value is obtained under the hypotheses of the existence of an opportune risk neutral probability measure in a complete market. The question of the incompleteness of the insurance product market, due to the demographic component, is fronted in several papers (see, for example, [2]). and solved introducing opportune probability measure.

We suppose that the variables $K_{x,t}$ are independent and identically distributed, the random variables F_j and *v(t,j)* are independent and identically distributed conditioning on the interest process, and the two risk sources $K_{x,t}$ and *v(t,j)* are independent.

§3 A Numerical Application

The application concerns a portfolio of homogeneous insured loan policies, issued to an unitary capital scheduled according to the amortization scheme at variable interest rates. At the end of the period in which the borrower-insured dies, the insurer pays the outstanding balance existing at the beginning of the unit time, plus the (variable) interest on this sum for the same period.

The refund occurs by means of a first payment instalment calculated according to a fixed contractual rate (cf., [6]). The following instalments are calculated on the basis of the Euribor rate observed the day before their maturity. According to our hypothesis, the instalments following the first one are uncertain, because their value is connected to the Euribor rate trend.

The first step is the estimation of the term structure of the Euribor rates in order to settle the succession of the future instalments. Then the evaluation of the reserve fair value by means of (1) follows, assuming as example of the best estimation for the cash flow fair valuation a term structure of interest rates based on the Cox-Ingersoll-Ross square root model and a mortality table constructed by means of the Lee Carter model. The behaviour of the portfolio fair reserve will be illustrated and the comparison between this trend with the case of a portfolio of insured loans in the case of amortization at fixed rate is showed. Other analysis and illustration will be proposed for studying interesting aspects of the contract in the perspective of the portfolio fair valuation.

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