ECONOMIC CAPITAL MANAGEMENT FOR INSURANCE COMPANIES USING CONDITIONAL VALUE AT RISK AND A COPULA APPROACH

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1 Introduction.

In this paper, we present a model for estimating the economic capital for non life insurance companies; the model takes into account real dependences amongst losses coming from different classes of insurance (CoI).

The main purpose is determining the diversification benefit considering a real dependence structure through a copula approach. We propose also the use of Conditional VaR (CVaR) to efficiently manage the economic capital of the company.

Our analysis is based on the calculation of loss ratios (LR) that represent the ratio of incurred claims and earned premium for a specified CoI.

We collected data from main Italian non life insurance companies for the period 1998-2004 and we built then semi annual loss ratios of a hypothetical representative insurance company for the following five classes of insurance: 1. Accident; 2. Land vehicles; 3. Goods in Transit ; 4. Fire and natural forces; 5. Motor vehicle liability.

We build for each CoI the loss ratio in the following way:

$$LR_{i,t} = \frac{IC_{i,t}}{EP_{i,t}} \qquad i = 1,\ldots,5$$

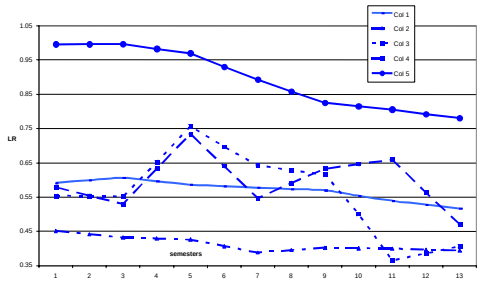
where $IC_{i,t}$ represents incurred claims and $EC_{i,t}$ represents earned premium during the period (t; t + 1) for the class of insurance i. We then define the aggregate loss ratio as $LR_t = \sum_{i=1}^{5} \xi_i \cdot LR_{i,t}$ where the weights ξ_i are all equal to 0.20; the choice of ξ_i has the aim of highlighting the role of correlations in the economic capital valuation; their sum is equal to one; in this way they represent the composition quotas of the earned premiums of the selected company.

As already said, the calculation of the economic capital of the company is based on the aggregate loss ratio of the company; it's obvious it should take into consideration correlations amongst the five CoIs.

At this purpose, we use a dependence structure given by copula functions. We compare results obtained with the Gaussian copula and the Student *t*-copula (with several degrees of freedom). We perform then a Monte Carlo simulation with *N* scenarios.

2 A Numerical Example.

The next graph shows the historical series of the semi annual loss ratios of each CoI in the described period (1998 – 2004).



Graph 1. *The historical series of loss ratios.*

We determine for each class of insurance the best fitting distribution using standard tools (such as @Risk). We found the following distributions:

CoI	μ	σ	skewness	κ	distribution	parameter 1	parameter 2
1	0.5707	0.0282	-0.6564	2.2195	logistic	0.5736	0.0158
2	0.4129	0.0206	0.6131	1.9537	pareto	16.6554	0.3884
3	0.5622	0.1211	-0.2755	2.0898	logistic	0.5683	0.0687
4	0.5984	0.0682	0.0783	2.7571	ext value	0.5658	0.0627
5	0.8953	0.0865	-0.0112	1.3307	exponential	0.1155	

Table 1. *The best fitting distributions for the loss ratios of the five CoIs.*

The empirical correlation matrix of the CoIs may be estimated using the historical series of the LRs; we obtained the following correlation table:

ρ	1	2	3	4	5
1	100%	71%	73%	20%	92%
2	71%	100%	26%	8%	85%
3	73%	26%	100%	45%	61%
4	20%	8%	45%	100%	12%
5	92%	85%	61%	12%	100%

Table 2. *The empirical correlation matrix for the loss ratios of the five CoIs.*

Nevertheless, empirical correlations are not reliable in general, due to the few data we have at our disposal. Consequently, we decided to compare empirical correlations with the "logical" values of the coefficients we previously described (section 1) to eliminate some false correlations.

This comparison deals to the following matrix:

ρ	1	2	3	4	5
1	100%	0%	0%	20%	75%
2	0%	100%	0%	0%	45%
3	0%	0%	100%	0%	45%
4	20%	0%	0%	100%	0%
5	75%	45%	45%	0%	100%

Table 3. *The coherent correlation matrix for the loss ratios of the five CoI.*

The degrees of freedom υ (that individuate the best Student Copula) are calculated with a log-likelihood estimation that gives $\upsilon = 3$.

To quantify the capital requirement and the diversification benefit we perform a Monte Carlo simulation with 100.000 scenarios. We consider the Gaussian copula and the Student copula with 10, 3 and 1 degrees of freedom respectively. The degree of freedom of the Student copula influences the tail dependence of the distribution. A low degree of freedom leads to high tail dependence while if we consider $\upsilon \to \infty$, the Student copula tends to the Gaussian copula, which has no tail dependence. The student copula with one degree of freedom is also called the Cauchy copula. Results coming from the adoption of the different copula dependence structures are then compared with results coming from the independence and the comonotonic assumptions.

The following table shows the results obtained performing the Monte Carlo simulation.

	Indep.	Gauss	Student t10	Student t3	Cauchy	Sum
mean	0.6090	0.6093	0.6080	0.6080	0.6087	
std.	0.0389	0.0481	0.0474	0.0480	0.0482	
skewness	0.5791	0.7927	0.8786	1.1140	1.4892	
kurtosis	4.3377	4.7055	5.1431	6.6535	8.8490	
VaR 95%	0.6774	0.6958	0.6937	0.6938	0.6955	0.7442
CVaR 95%	0.7027	0.7312	0.7286	0.7357	0.7449	0.8027
VaR 99%	0.7178	0.7534	0.7504	0.7596	0.7747	0.8383
CVaR 99%	0.7429	0.7872	0.7865	0.8060	0.8271	0.8979

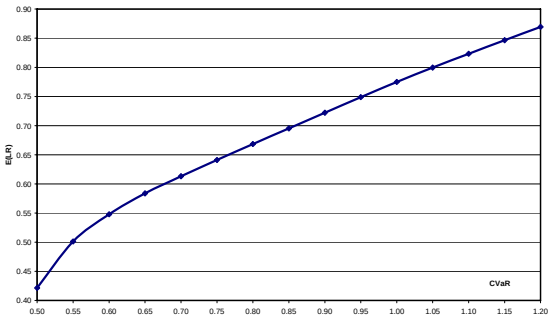
Table 4: *Statistics and risk measures of the company using several copula functions.*

Obviously, the diversification benefit is the difference between results of the various copulas with those deriving from the comonotonic assumptions.

Finally, we built the efficient frontier in the plane LR – CVaR.

This is an optimisation problem that consists in maximizing the expected aggregate loss ratio (varying the weights ξ_i, which must sum up to one) with a fixed conditional CVaR level τ.

We then represent graphically the efficient frontier:



Graph 2: *The efficient frontier LR - CVaR.*

The efficient frontier may be used to manage the total risk of the company and to individuate the best composition of the insurance business amongst the various CoIs line in correspondence of a specified economic capital requirement (CVaR).

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